Advection-Dominated Accretion Disks: Geometrically Slim or Thick?

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Abstract

We revisit the vertical structure of black hole accretion disks in spherical coordinates. By comparing the advective cooling with the viscous heating, we show that advection-dominated disks are geometrically thick, i.e., with the half-opening angle $\Delta\theta > 2\pi/5$, rather than slim as supposed previously in the literature.

Key words: accretion, accretion disks — black hole physics — hydrodynamics

1. Introduction

It was known long since that the very basic assumption of the Shakura-Sunyaev disk (SSD, Shakura & Sunyaev 1973), that is, the geometrical thinness of the disk, $H/R \ll 1$, where H is the half thickness of the disk and R is the radius in cylindrical coordinates, would break down for the inner region of the disk in some specific situations. For example, when the mass accretion rate $\dot{M}$ approaches and surpasses its critical value corresponding to the Eddington luminosity, radiation pressure will act to huff the inner region of the disk in the vertical direction; or when the cooling mechanism is inefficient, so that the temperature in the disk becomes very high, then gas pressure will act in a similar way. In either of these two situations, the inner region of the disk will get geometrically thick, i.e., with $H/R \sim 1$ (e.g., Frank et al. 2002, p.98). Based on these understandings, two types of models were proposed more than twenty years ago, namely the optically thick, radiation pressure-supported thick disk (Abramowicz et al. 1978; Paczyński & Wiita 1980; Madau 1988) and the optically thin, ion pressure-supported thick disk (Rees et al. 1982). To avoid mathematical difficulties, in these models the disk was assumed to be purely rotating, i.e., with no mass accretion. However, the very existence of non-accreting thick disks was thrown into doubt by the discovery of Papaloizou & Pringle (1984) that such disks are dynamically unstable to global non-axisymmetric modes. Since the work of

Blaes (1987), it had been recognized that it is accretion, i.e., radial matter motion and energy advection into the central black hole, that can sufficiently stabilize all modes. Accordingly, the concept of advection dominance was introduced and two new types of models were constructed, namely the optically thick, radiation pressure-supported slim disk (Abramowicz et al. 1988) and the optically thin, ion pressure-supported, advection-dominated accretion flow (ADAF, Narayan & Yi 1994; Abramowicz et al. 1995). Both these two types of models are popular nowadays.

Slim disks and ADAFs were supposed to be geometrically slim, i.e., with $H/R \lesssim 1$, neither thin nor thick. The reason for this restriction is the following. As argued by Abramowicz et al. (1995), the advection factor $f_{\rm adv} \equiv Q_{\rm adv}/Q_{\rm vis}$, where $Q_{\rm adv}$ is the advective cooling rate per unit area and $Q_{\rm vis}$ is the viscous heating rate per unit area, should satisfy the relation

$$f_{\rm adv} \gtrsim \left(\frac{H}{R}\right)^2 . \tag{1}$$

Obviously, advection can be important only for disks that are not thin. But the disk cannot be thick either, because the value of $f_{\rm adv}$ cannot exceed 1.

Recently, Gu & Lu (2007, hereafter GL07) addressed a problem in the slim disk model of Abramowicz et al. (1988, see also Kato et al. 1998). In this model, the gravitational potential was approximated in the form suggested by Hōshi (1977), i.e.,

$$\psi(R,z) \simeq \psi(R,0) + \frac{1}{2}\Omega_{\rm K}^2 z^2 , \tag{2}$$

where $\Omega_{\rm K}$ is the Keplerian angular velocity. As shown by GL07, such an approximation is valid only for geometrically thin disks with $H/R \lesssim 0.2$, and for a larger thickness it would greatly magnify the gravitational force in the vertical direction. Accordingly, the widely adopted relationship $H\Omega_{\rm K}/c_s =$ constant can approximately hold only for thin disks as well. Since formula (1) was derived by using this relationship, its validity for thicker disks has not been justified. GL07 noted that, when the vertical gravitational force is correctly calculated with the explicit potential $\psi(R,z)$, "slim" disks are much thicker than previously thought. However, the work of GL07 was still within the framework of the slim disk model in some sense. In particular, those authors did not consider the vertical distribution of velocities, but instead kept the assumption of vertical hydrostatic equilibrium,

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\partial \psi}{\partial z} = 0 , \tag{3}$$

which is a simplification of the more general vertical momentum equation

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\partial \psi}{\partial z} + v_R\frac{\partial v_z}{\partial R} + v_z\frac{\partial v_z}{\partial z} = 0 \tag{4}$$

(e.g., Abramowicz et al. 1997), where ρ is the mass density, p is the pressure, and v_R and v_z are the cylindrical radial and vertical velocities, respectively. While the terms containing v_z in equation (4) can be reasonably dropped for thin disks because in this case v_z must be

negligibly small, it needs a careful consideration whether the same can be done for not thin disks (Abramowicz et al. 1997, also see below in §2).

Also regarding to the two main features of advection-dominated disks, i.e., the advection dominance and the slimness, an important different approach was made earlier by Narayan & Yi (1995, hereafter NY95). NY95 considered rotating spherical accretion flows ranging from the equatorial plane to the rotation axis, i.e., with $H/R \to \infty$ and with no free surfaces. They assumed self-similarity in the radial direction and solved differential equations describing the vertical structure of the flow, and showed that, comparing to their exact solutions, the solutions obtained previously with the vertical integration approach are very good approximations, provided "vertical" means the spherical polar angle θ, rather than the cylindrical height z. This seemed to indicate that advection-dominated disks are not necessarily limited to be slim. However, those authors did not calculate the advection factor $f'_{\rm adv}$ (they defined $f'_{\rm adv} \equiv q_{\rm adv}/q_{\rm vis}$, with $q_{\rm adv}$ and $q_{\rm vis}$ being the advective cooling rate and the viscous heating rate per unit volume, respectively), but rather set it a priori to be a constant. It is still not answered how their $f'_{\rm adv}$ varies with θ, or how $f_{\rm adv}$ per unit area varies with the thickness of the disk, and what is required for advection to be dominant.

In this work we try to make some complementarity to NY95 and some refinements to GL07. We consider the vertical structure of accretion flows with free surfaces and show that advection-dominated disks must be geometrically thick rather than slim. Our results may suggest to recall the historical thick disk models mentioned above, but with improvements that they have to include accretion now.

2. Equations

We consider a steady state axisymmetric accretion flow in spherical coordinates (r, θ, ϕ) and use the Newtonian potential $\psi = -GM/r$ since it is convenient for the self-similar formalization adopted below, where M is the black hole mass. The basic equations of continuity and momenta are

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \rho v_\theta) = 0 \ , \tag{5}$$

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\left(\frac{\partial v_r}{\partial\theta} - v_\theta\right) - \frac{v_\phi^2}{r} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} \ , \tag{6}$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\left(\frac{\partial v_\theta}{\partial\theta} + v_r\right) - \frac{v_\phi^2}{r}\cot\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} \ , \tag{7}$$

$$v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial\theta} + \frac{v_\phi}{r}(v_r + v_\theta\cot\theta) = \frac{1}{\rho r^3}\frac{\partial}{\partial r}(r^3 t_{r\phi}) \tag{8}$$

(e.g., Xue & Wang 2005), where v_r, v_θ, and v_ϕ are the three velocity components. We assume that only the $r\phi$-component of the viscous stress tensor is important, which is $t_{r\phi} = \nu\rho r \partial(v_\phi/r)/\partial r$, where $\nu = \alpha c_s^2 r/v_{\rm K}$ is the kinematic viscosity coefficient, α is the con-

stant viscosity parameter, c_s is the sound speed defined as $c_s^2 = p/\rho$, and $v_{\rm K} = (GM/r)^{1/2}$ is the Keplerian velocity.

We do not simply assume vertical hydrostatic equilibrium (eq. [3]). Equation (7) is the general vertical momentum equation in spherical coordinates, corresponding to equation (4) in cylindrical coordinates. Abramowicz et al. (1997) have given several reasons why spherical coordinates are a much better choice. We only mention one of these reasons that is particularly important for our study here. The stationary accretion disks calculated in realistic two-dimensional (2D) and three-dimensional (3D) simulations resemble quasi-spherical flows, i.e., in spherical coordinates the half-opening angle of the flow $\Delta\theta \approx$ constant, or in cylindrical coordinates the relative thickness $H/R \approx$ constant, much more than quasi-horizontal flows, i.e., $H \approx$ constant (e.g., Papaloizou & Szuszkiewicz 1994; NY95). If no outflow production from the surface of the disk is assumed, then obviously $v_\theta = 0$ is a reasonable approximation for disks with any thickness (Xue & Wang 2005); but v_z cannot be neglected for not thin disks because there is a relation $v_z/v_R \sim H/R$ for quasi-spherical flows, making equation (4) difficult to deal with.

Similar to NY95, we assume self-similarity in the radial direction

$$v_r \propto r^{-1/2};\ v_\theta = 0;\ v_\phi \propto r^{-1/2};$$
$$\rho \propto r^{-3/2};\ c_s \propto r^{-1/2}.$$

The above relation automatically satisfies the continuity equation (5). By substituting the relation, the momentum equations (6-8) are reduced to be

$$\frac{1}{2}v_r^2 + \frac{5}{2}c_s^2 + v_\phi^2 - v_{\rm K}^2 = 0 \ , \tag{9}$$

$$\frac{c_s^2}{p}\frac{dp}{d\theta} = v_\phi^2 \cot\theta \ , \tag{10}$$

$$v_r = -\frac{3}{2}\frac{\alpha c_s^2}{v_{\rm K}} \ . \tag{11}$$

Four unknown quantities, namely v_r, v_ϕ, c_s and p, appear in these three equations. This is because we do not write the energy equation, whose general form is $q_{\rm vis} = q_{\rm adv} + q_{\rm rad}$, where $q_{\rm rad}$ is the radiative cooling rate per unit volume. In principle, the general energy equation should be solved, and then $f'_{\rm adv}$ is obtained as a variable, as done, e.g., by Manmoto et al. (1997) for ADAFs and by Abramowicz et al. (1988) and Watarai et al. (2000) for slim disks. But due to complications in calculating the radiation processes, in NY95 and even in works on global ADAF solutions (e.g., Narayan et al. 1997), $q_{\rm adv} = f'_{\rm adv} q_{\rm vis}$ or $Q_{\rm adv} = f_{\rm adv} Q_{\rm vis}$ was used instead as an energy equation and $f'_{\rm adv}$ or $f_{\rm adv}$ was given as a constant. Since our purpose here is to investigate the variation of $f_{\rm adv}$ with the thickness of the disk, we wish to calculate $Q_{\rm adv}$ and $Q_{\rm vis}$ respectively, and then estimate $f_{\rm adv}$. To do this, we further assume a polytropic relation, $p = K\rho^\gamma$, in the vertical direction, which is often adopted in the vertically integrated models of geometrically slim disks (e.g., Kato et al. 1998, p.241). We admit that the polytropic

assumption is a simple way to close the system, and then enables us to calculate the dynamical quantities and evaluate $f_{\rm adv}$ self-consistently.

With the polytropic relation and the definition of the sound speed $c_s^2 = p/\rho$, equation (10) becomes

$$\frac{dc_s^2}{d\theta} = \frac{\gamma - 1}{\gamma} v_\phi^2 \cot\theta \ , \tag{12}$$

which along with equations (9) and (11) can be solved for v_r, v_ϕ, and c_s. A boundary condition is required for solving the differential equation (12), which is set to be $c_s = 0$ (accordingly $\rho = 0$ and $p = 0$) at the surface of the disk. The quantities $q_{\rm adv} = pv_r(\partial \ln p/\partial r - \gamma \partial \ln \rho/\partial r)/(\gamma - 1)$ and $q_{\rm vis} = \nu\rho r^2[\partial(v_\phi/r)/\partial r]^2$ are expressed in the self-similar formalism as

$$q_{\rm adv} = -\frac{5 - 3\gamma}{2(\gamma - 1)} \frac{pv_r}{r} \ , \tag{13}$$

$$q_{\rm vis} = \frac{9}{4} \frac{\alpha p v_\phi^2}{r v_{\rm K}} \ , \tag{14}$$

then $Q_{\rm adv}$ and $Q_{\rm vis}$ are given by the vertical integration,

$$Q_{\rm adv} = \int_{\frac{\pi}{2}-\Delta\theta}^{\frac{\pi}{2}+\Delta\theta} q_{\rm adv} \ r \sin\theta \ d\theta \ , \tag{15}$$

$$Q_{\rm vis} = \int_{\frac{\pi}{2}-\Delta\theta}^{\frac{\pi}{2}+\Delta\theta} q_{\rm vis} \ r \sin\theta \ d\theta \ , \tag{16}$$

and $f_{\rm adv} \equiv Q_{\rm adv}/Q_{\rm vis}$ is obtained. In our calculations $\alpha = 0.1$ is fixed.

3. Numerical results

We first study the variation of dynamical quantities with the polar angle θ for a given disk's half-opening angle $\Delta\theta$. Figure 1 shows the profiles of v_r (the dashed line), v_ϕ (the dot-dashed line), c_s (the solid line), and ρ (the dotted line) for three pairs of parameters, i.e., $\gamma = 4/3$ and $\Delta\theta = 0.25\pi$ for Fig. 1*a*, $\gamma = 4/3$ and $\Delta\theta = 0.45\pi$ for Fig. 1*b*, and $\gamma = 1.65$ and $\Delta\theta = 0.498\pi$ for Fig. 1*c*. The parameters are marked in Figure 3 by filled stars, which clearly show the corresponding values of the advection factor $f_{\rm adv}$. Obviously, advection is not significant for case *a* ($f_{\rm adv} < 0.1$), but is dominant for cases *b* and *c* ($0.5 < f_{\rm adv} < 1$). Comparing our results with Fig. 1 of NY95, it is seen that the profiles of v_r and ρ are similar, i.e., v_r (the absolute value) and ρ increase with increasing θ and achieve the maximal value at the equatorial plane ($\theta = \pi/2$). On the contrary, the two profiles of c_s are significantly different. In their Fig. 1, the value of c_s decreases with increasing θ and achieves the minimal value at the equatorial plane; in our Fig. 1, however, c_s increases with increasing θ and achieves the maximal value at the equatorial plane. In our opinion, the difference results from different assumptions, i.e., NY95 assumed an energy advection factor $f'_{\rm adv}$ in advance, whereas we solve for the energy advection factor $f_{\rm adv}$ self-consistently based on a polytropic relation in the vertical direction. We think that our profile for c_s is reasonable for disk-like accretion. For example, in the standard thin

disk, the direction of the radiative flux is from the equatorial plane to the surface, which means that the temperature (or the sound speed) decreases from the equatorial plane to the surface. Such a picture agrees with our Fig. 1 but conflicts with Fig. 1 of NY95.

Figure 2 shows the variation of $f_{\rm adv}$ with $\Delta\theta$ for the ratio of specific heats $\gamma = 4/3$. Advection dominance means $0.5 < f_{\rm adv} \leq 1$. We first explain the two dashed lines and the dotted line that correspond to previous works in the slim disk model, then the solid line that represents our results here, and leave the dot-dashed line later. Both the two dashed lines are obtained by assuming vertical hydrostatic equilibrium (eq. [3]) and using the Hōshi form of potential (eq. [2]), thus the relation $H\Omega_{\rm K}/c_s =$ constant is adopted. The difference between these two lines is the following. For line a, the simple one-zone treatment in the vertical direction is made as in the SSD model; then in equation (3), $\partial p/\partial z \approx -p/H$, $\partial\psi/\partial z \approx \Omega_{\rm K}^2 H$, and $H\Omega_{\rm K}/c_s = 1$ is obtained (e.g., Kato et al. 1998, p.80). For line b, there is some improvement in the sense that the vertical structure of the disk is considered. By assuming a polytropic relation, the vertical integration of equation (3) gives $H\Omega_{\rm K}/c_s = 3$ (e.g., Kato et al. 1998, p.242). Because of these different treatments in the vertical direction, these two lines show different variations of $f_{\rm adv}$ with $\Delta\theta$ and different maximum values of $\Delta\theta$. The upper limit of $f_{\rm adv}$ is 1 (full advection dominance), beyond which there would be no thermal equilibrium solutions. It can be analytically derived that for the case of line a, the maximum value of $\Delta\theta$ corresponding to $f_{\rm adv} = 1$ is $\Delta\theta_{\rm max} = \arctan(\sqrt{2/7})$, or in cylindrical coordinates the maximum relative thickness $(H/R)_{\rm max} = \sqrt{2/7}$; and for the case of line b it is $\Delta\theta_{\rm max} = \arctan(3/2)$ or $(H/R)_{\rm max} = 3/2$. As mentioned in §1, the thickness of the disk in the slim disk model had been underestimated because the vertical gravitational force was overestimated by the Hōshi form of potential. Even so, according to the more sophisticated version of the slim disk model (line b), advection dominance $f_{\rm adv} > 0.5$ would require $H/R > 1$ ($\Delta\theta > \pi/4$), and full advection dominance would require $H/R = 3/2$, in contradiction with $H/R \lesssim 1$, the supposed feature of the model.

The dotted line in Figure 2 is for the results of GL07. The point made in that work was that the explicit potential $\psi(R, z)$, rather than its Hōshi approximation (eq. [2]), was used, so that the vertical gravitational force was correctly calculated. But GL07 still kept the assumption of vertical hydrostatic equilibrium (eq. [3]), i.e., the terms containing v_z in equation (4) were incorrectly ignored. Because of this, the thickness of the disk was overestimated; and accordingly, it seemed that advection dominance can never be possible, since even for the extreme thickness $\Delta\theta = \pi/2$ (or $H/R \to \infty$) the value of $f_{\rm adv}$ can only marginally reach to 0.5.

We make improvements over GL07. We use spherical coordinates with the assumption $v_\theta = 0$, which is better than $v_z = 0$ in cylindrical coordinates; and then calculate the vertical distribution of velocities (v_r and v_ϕ) and thermal quantities (ρ, p, and c_s). Our results are shown by the solid line in Figure 2. It is seen that advection dominance ($f_{\rm adv} > 0.5$) is possible, but only for $\Delta\theta > 2\pi/5$ (or 72°). Therefore, advection-dominated disks must be geometrically

thick, rather than slim as previously supposed.

It is also seen that line b, the dotted line, and the solid line in Figure 2 almost coincide with each other for thin disks with $\Delta\theta \lesssim 0.1\pi$. This is natural, since for thin disks both the Hōshi approximation of potential and the assumption of vertical hydrostatic equilibrium are valid, and the three approaches represented by the three lines make no significant difference. But the one-zone treatment, i.e., total ignorance of the vertical structure of the disk, seems to be too crude, making the resulting line a deviate from the other three lines even for thin disks.

The value $\gamma = 4/3$ in Figure 2 corresponds to the optically thick and radiation pressure-dominated case, to which the historical radiation pressure-supported thick disk and the slim disk belong; while it is $\gamma \to 5/3$ for the optically thin and gas pressure-dominated case, to which the historical ion pressure-supported thick disk and the ADAF belong. In Figure 3, the four solid lines show variations of $\Delta\theta$ with γ for four given values of $f_{\rm adv}$. It is seen that advection dominance ($f_{\rm adv} > 0.5$) requires $\Delta\theta$ to be large for any value of γ; and that for a fixed $f_{\rm adv}$ (the same degree of advection), the required $\Delta\theta$ increases with increasing γ, that is, for advection to be dominant, optically thin disks must get even geometrically thicker than optically thick ones.

For the geometrically thin case, $\Delta\theta \ll 1$, the Taylor expansion of equations (9), (11), and (12) with respect to $\Delta\theta$ can be performed, and we derive an approximate analytic relation:

$$f_{\rm adv} \approx \frac{(5-3\gamma)(2\gamma-1)}{3\gamma(5\gamma-3)} \cdot \Delta\theta^2 \ , \tag{17}$$

which is similar to equation (1) in cylindrical coordinates. The dot-dashed lines in Figures 2 and 3 correspond to equation (17) for a fixed $\gamma = 4/3$ and for a fixed $f_{\rm adv} = 0.01$, respectively. It is seen from Figure 2 that, as expected, the analytic approximation of equation (17) agrees well with the correct numerical results (the solid line) for small $\Delta\theta$, but deviates a lot for large $\Delta\theta$. In Figure 3 a good agreement between equation (17) and the numerical results (the lowest solid line) is seen again, especially for small values of γ. The limitation that equation (17) is valid only for small $\Delta\theta$, and accordingly only for small $f_{\rm adv}$, should also apply to equation (1), because that equation is derived with the Hōshi form of potential.

4. Discussion

The key concept of the slim and ADAF disk models is advection dominance. This concept was introduced rather as an assumption, whether and under what physical conditions can it be realized have not been clarified. The main result of our work is to have shown that, in order for advection to be dominant, the disk must be geometrically thick with the half-opening angle $\Delta\theta > 2\pi/5$, rather than slim as suggested previously in the slim disk and ADAF models. Thus, advection-dominated disks are geometrically similar to the historical thick disks metioned in §1. This result is obvious because, as revealed in GL07, in the slim disk and ADAF models the vertical gravitational force was overestimated by using the Hōshi's approximate potential, and

accordingly the disk's thickness was underestimated. NY95 considered accretion flows with no free surfaces and found that when the given advective factor $f'_{\rm adv}(\equiv q_{\rm adv}/q_{\rm vis}) \to 1$ (full advection dominance), their solutions approach nearly spherical accretion. If "nearly spherical" can be regarded as extremely thick, then their results and ours agree with each other, but we take a different approach. We do not give the value of $f_{\rm adv}(\equiv Q_{\rm adv}/Q_{\rm vis})$ in advance, but instead consider accretion flows with free surfaces, i.e., accretion disks. The boundary condition is set to be $p = 0$, which is usually adopted in the literature (e.g., Kato et al. 1998). Then the thickness of the disk, $\Delta\theta$, makes sense, and we calculate $f_{\rm adv}$ to see how it relates to $\Delta\theta$.

Many 2D and 3D numerical simulations of viscous radiatively inefficient accretion flows (RIAFs) revealed the existence of convection-dominated accretion flows (CDAFs), while ADAFs could not be obtained (e.g., Stone et al. 1999; Igumenshchev & Abramowicz 2000; McKinney & Gammie 2002; Igumenshchev et al. 2003). We think that this fact probably indicates that the existing analytic ADAF models might have hidden inconsistencies, and the incorrect treatment of the vertical structure might be one such inconsistency, as addressed in our work. Moreover, the recent radiation-MHD simulations (Ohsuga et al. 2009) showed that the disk is geometrically thick in their models A and C (corresponding to slim disks and ADAFs, respectively), which is in agreement with our results.

Apart from the convective motion, the outflow is found in 2D and 3D MHD simulations of non-radiative accretion flows (e.g., Stone & Pringle 2001; Hawley & Balbus 2002). For optically thick flows, the circular motion and the outflow are found in 2D radiation-HD simulations (e.g., Ohsuga et al. 2005; Ohsuga 2006). The assumption $v_\theta = 0$ would break down when the convective motion or the outflowing motion is significant, thus we have to point out the limitation of our solutions, which are based on the self-similar assumption in the radial direction and particularly for $v_\theta = 0$.

In this paper we have not shown the exact thermal equilibrium solution for a certain mass accretion rate. We wish to stress that our main concern here is the relationship between the energy advection factor and the thickness of the disk. The well-known formula (1), which was previously believed to be valid for both optically thick and thin disks, implied that advection-dominated accretion disks are geometrically slim. As shown in Figures 2 and 3, however, formula (1) is inaccurate for disks that are not geometrically thin. We think that the new relationship between $f_{\rm adv}$ and $\Delta\theta$, shown in Figures 2 and 3, should also work for both optically thick and thin cases. Even without the exact solutions, we can predict that advection-dominated accretion disks ought to be geometrically thick rather than slim. Our next work will concentrate on the optically thick disks and take the radiative cooling into consideration. In the vertical direction, we will solve the dynamical equations combined with the radiative transfer equations, thus the polytropic assumption will be relaxed. At that step, we will be able to calculate the thermal equilibrium solutions with given mass accretion rates and show the optical depth, pressure, and luminosity of the disks.

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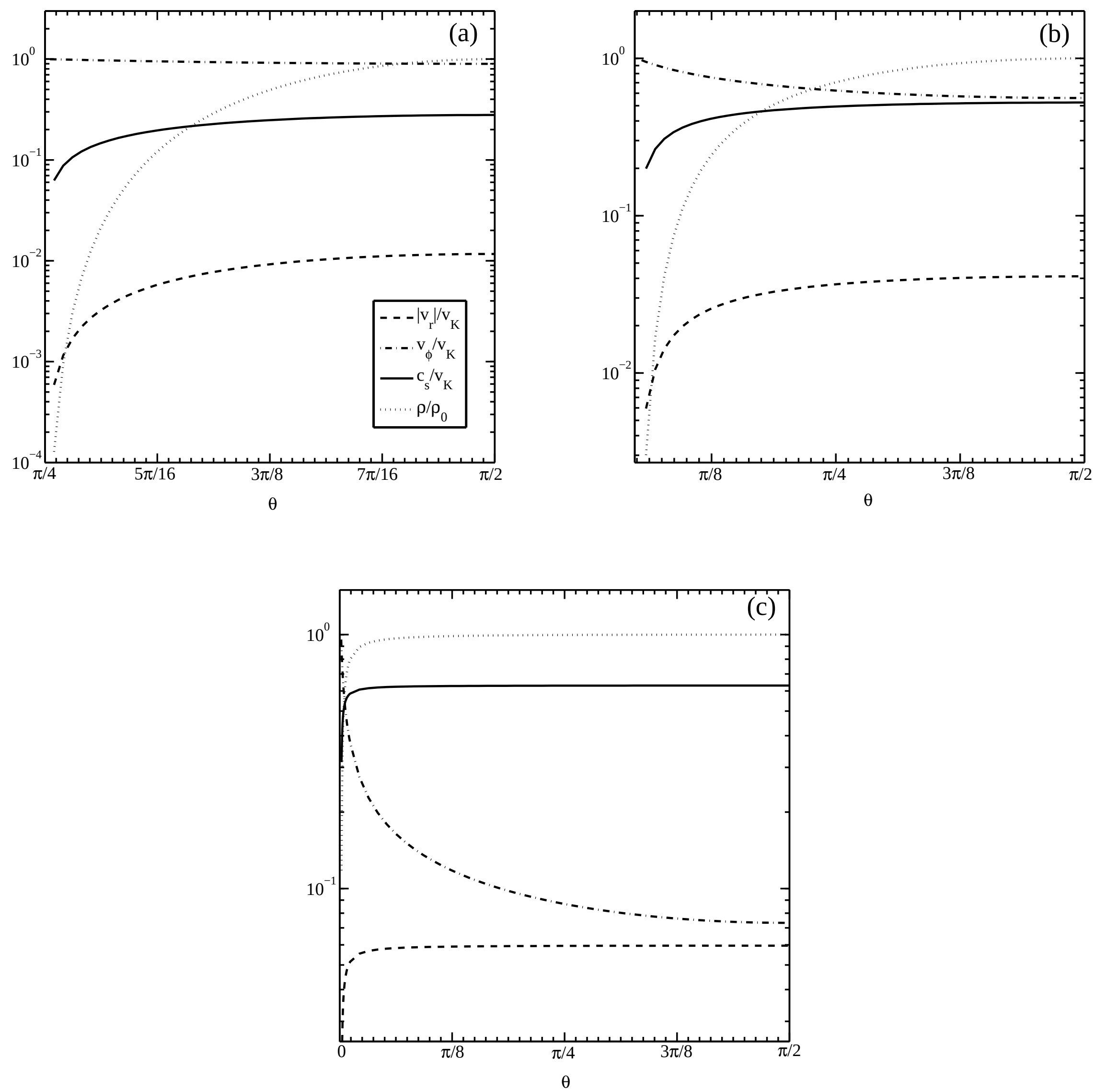

Fig. 1. Variations of v_r, v_ϕ, c_s, and ρ with the polar angle θ for three pairs of parameters: (a) $\gamma = 4/3$ and $\Delta\theta = 0.25\pi$; (b) $\gamma = 4/3$ and $\Delta\theta = 0.45\pi$; (c) $\gamma = 1.65$ and $\Delta\theta = 0.498\pi$.

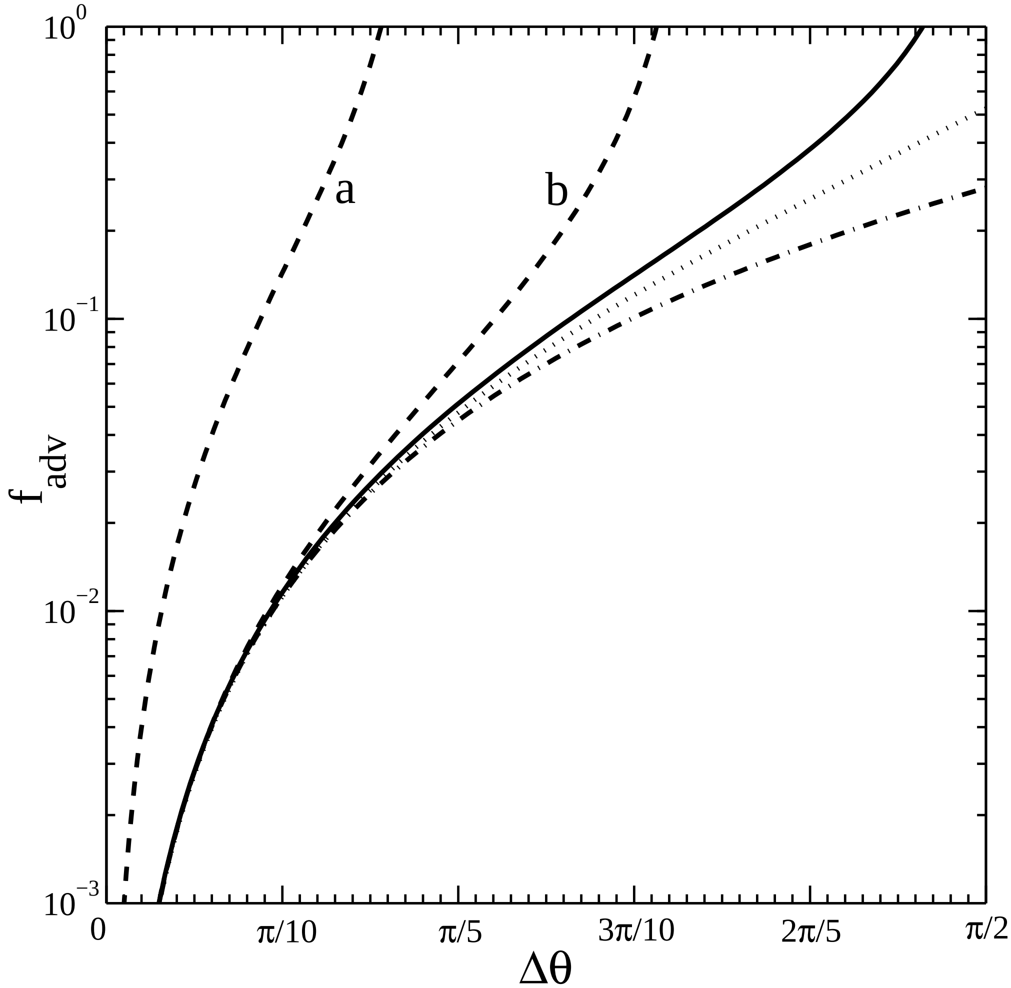

Fig. 2. Variation of the advection factor $f_{\rm adv}$ with the disk's half-opening angle $\Delta\theta$ for the ratio of specific heats $\gamma = 4/3$. The solid line shows our numerical results. The dot-dashed line corresponds to the analytic approximation of equation (17). The two dashed lines are for the previous results in the slim disk model with the Hōshi form of potential, and the dotted line is for the previous results in GL07.

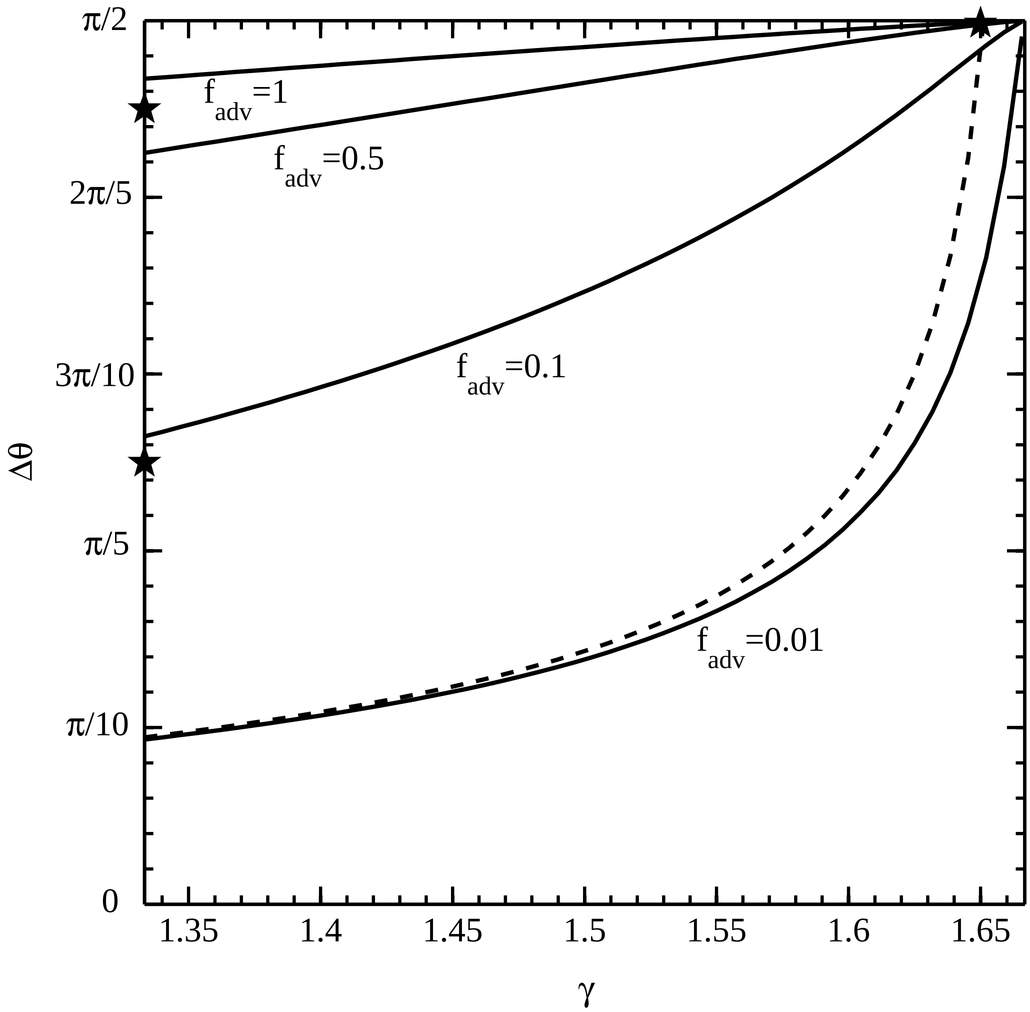

Fig. 3. Variation of $\Delta\theta$ with γ for given values of $f_{\rm adv}$. The solid lines show numerical results, and the dot-dashed line corresponds to equation (17). The three filled stars denote the parameters chosen in Figure 1.